Annals of Finance (2006) 2: 397–405
DOI 10.1007/s10436-006-0043-1

RESEARCH ARTICLE

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Generalised rational bias in financial forecasts

Received: 17 October 2005 / Accepted: 27 March 2006 / Published online: 4 May 2006

Abstract Laster et al. (Q J Econ 114(1):293–318, 1999) built an economic model in which forecasters have incentives to generate forecasts that differ form the consensus. It is shown that the dispersion of the equilibrium distribution of forecasters, depends on the relative importance given on the intensive forecast users' loss versus the publicity gain from occasional users. These results depend heavily on the assumption of symmetry for the loss and density functions. In this paper we examine the effects of generalising loss preferences and probability densities to allow for asymmetries through the LinEx loss and the Skewed Normal density, respectively. We derive the generalised equilibrium distribution of forecasts which contains the results of Laster et al. as a special case. The presence of asymmetric preferences is shown to cause a movement of the distribution away from the conditional mean, towards the optimal forecast under loss asymmetry. Furthermore, forecasts now tend to cluster around this quantity in an asymmetric way. These effects tend to be further strengthened or partially offset by the presence of skewness in the distribution of data, a result consistent with the conclusions of Christodoulakis (Finan Res Lett 2:227–233, 2005).

Keywords Asymmetric preferences · Distribution of forecasters · LinEx · Non-normality · Optimal forecasts · Skewed normal

The author is grateful to an anonymous referee for helpful comments that have improved the paper. The views expressed in the paper are those of the author and should in no part be attributed to the Bank of Greece.

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JEL Classification Numbers G1 · C5

1 Introduction

Optimal forecasting which deviates form the rationally expected conditional mean of the distribution of data has long been justified by Granger (1969) using asymmetric loss functions. In a recent paper, Laster et al. (1999) have built an economic model to explain the possibility of rational bias in optimal forecasts and establish economic motivation for a representative rational forecaster to generate forecasts that are away from the mean of the distribution. In an economy with N forecasting firms, the forecasters act so as to maximise their compensation as a function of the intensive forecast users' loss function and the publicity gain of the firm generated from occasional users. In the presence of both types of forecast users, ceteris paribus, Laster et al. (1999) show that the dispersion of the equilibrium distribution of forecasters depends on the relative importance given to the intensive forecast users' loss versus the publicity gain. That is, the higher the weight on the forecast user loss, the higher the concentration of the forecasters around the mean of the distribution of the variable under prediction. These results depend heavily on the assumption of symmetric quadratic loss preferences for the forecast user and symmetric density for the variable under investigation.

In this paper we examine the effects of generalising loss preferences and probability densities to allow for asymmetries through the LinEx loss, see Varian (1974) and the Skewed Normal density, see O'Hagan and Leonard (1976) and Arnold and Lin (2004), respectively. The optimal forecasting under asymmetric loss functions has been first addressed by Granger (1969) and further studied by Christoffersen and Diebold (1997) whilst its interaction with non-normality has been addressed in Christodoulakis (2005). It is known that the presence of asymmetric loss is sufficient to generate a rational bias away from the conditional mean. Laster et al. (1999) offer an alternative interpretation which maintains the loss function symmetry and in this paper we study the effects of the presence of loss and density asymmetry on their model. Asymmetric preferences are shown to cause an asymmetric movement of the equilibrium distribution of the forecasters away from the conditional mean, which is further strengthened or partially offset by the presence of skewness in the distribution of data. The paper is organised as follows: in section 2 we generalise the theory of Laster et al. (1999) for asymmetric loss and densities, in section 3 we discuss the implications of our results for the trade-off between model parameters and link our model to empirical examples on earnings forecasts and in section 4 we present our concluding remarks.

2 Generalised optimal forecasting

Laster et al. (1999) considered a one-period economy in which N forecasting firms operate to predict a macroeconomic variable y. We shall adopt their model in the context of financial forecasting, e.g. the analysts' earnings forecasts. At date $t = 0$ the analysts announce their forecast for date $t = 1$ using common information on the distribution of earnings y and the distribution of N forecasts. The assumption of common knowledge of the data probability density function is not very realistic. However, it allows the model to generate forecast heterogeneity due to preferences, without appealing to differences in the assessment of the likelihood of data. Furthermore, the assumption of knowledge of the contemporaneous distribution of forecasts intends to reflect an active analysts industry whose members reveal their views and compete openly in the market. At date $t = 1$ the forecast error becomes observable and the forecaster's wage is set based on ex-post performance, as a function of the intensive forecast users' loss and the publicity gain from occasional forecast users. Intensive users may represent active portfolio managers who would experience larger losses from earnings over-prediction relative to equal under-prediction, whilst occasional users could be strategic or high-net-worth asset allocators.

Consider a stochastic process $\{y_t\}$ adapted to a filtration I_t on a probability space $(\Omega, \Im, P)$. In what follows, $E_t = E(\cdot\,|I_t)$ denotes the conditional expectation operator. Thus, the forecasters will choose the forecast $\hat{y}$ that maximises the expected wage w

$$\begin{aligned} &\underset{\hat{y}}{\operatorname{argmax}}\, E_t\left(w\left(\hat{y}\right)\right) = v - sE_tL\left(y-\hat{y}\right) + bE_tA\left(y\right) \\ &A\left(y\right) = \left\{\begin{array}{cc} \frac{P}{n(y)} & \text{if } \hat{y} = y \\ 0 & \text{otherwise} \end{array}\right\} \end{aligned} \tag{1}$$

where v, s and b are constants, $L(.)$ is the intensive forecast users' loss function, $E_tA\left(y\right) = [Pf\left(\hat{y}\right)]/n\left(\hat{y}\right)$ is the expected value of the forecaster's publicity bonus for correctly predicting y and $f(y)$, $n(y)$ denote the density function of y and the contemporaneous distribution of forecasts, respectively, and P is the monetary value of publicity to the firm. Laster et al. (1999) assumed the existence of a symmetric quadratic loss L and a symmetric density f and solved the objective function with respect to $n(y)$. Their conclusions were then drawn from its comparative statics for different levels of emphasis on loss relative to publicity (s/b).

In the following we shall generalise the Laster et al. results by allowing the existence of a LinEx loss function and non-normality captured by the Skew Normal distribution. A LinEx loss function is defined as

$$L\left(y-\hat{y}\right) = \frac{1}{a^2}\left(\exp\left(a\left(y-\hat{y}\right)\right) - a\left(y-\hat{y}\right) - 1\right)$$

where $a \in R\backslash\{0\}$ and penalises asymmetrically forecast errors depending on the sign and size of a. As $a \to 0$, the LinEx converges to the symmetric quadratic loss function. In forecasting theory, minimisation of expected loss uses all the information about the likelihood of the data and the loss preferences. It is possible to replace the information contained in the density function by a number called "certainty equivalent" which should lead to the same optimal forecast as under expected loss. In this case, optimal forecasts are independent of all higher moments beyond the mean, a result generally true for preferences that are quadratic or can be locally approximated as quadratic. Departures from certainty equivalence appear in the cases of asymmetric preferences and/or model non-linearity, see for example Nobay and Peel (2003), Ruge-Murcia (2004) and Dolado et al. (2003, 2005). In our context, LinEx preferences introduce asymmetries which determine optimal forecasts as a function of mean and variance (for normality) and even higher moments (for non-normality), a result that prevents analysis based on certainty equivalents. Although this constitutes a departure from standard theory, the relevance of asymmetric loss functions in financial decision making emerges from the asymmetrically risk-sensitive nature of financial decisions. For example, for a prudent decision maker, over-prediction (under-prediction) of future earnings or asset returns (volatility) may cause higher losses to a portfolio or project manager as compared to an equal under-prediction (over-prediction). Thus, optimal forecasts in this context adjust the standard certainty equivalence results by a preference-weighted function of higher moments, reflecting the prudence or imprudence of the forecaster.

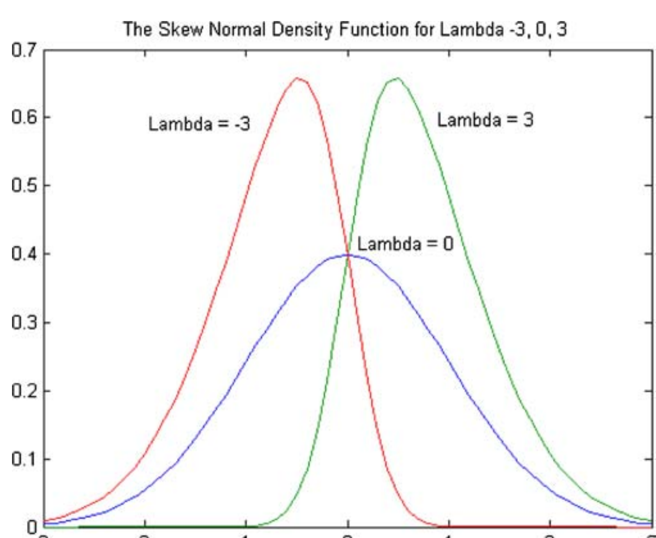
Let the data generating process be $y = \sigma v$ where $v \sim SN(\lambda)$, λ is a real constant and σ is volatility. Then, the density function of the Skew Normal distribution for the noise v is given by

$$\text{pdf}(v) = 2\varphi(v)\,\Phi(\lambda v)$$

where

$$\varphi(v) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v^2}{2}\right)$$

is the standard normal density and $\Phi(\ )$ is the cumulative standard normal. The Skew Normal accommodates a variety of skewness patterns as λ varies, whilst it converges to the Normal as $\lambda \to 0$. In Graph 1 we plot the Skew Normal density for $\lambda = -3, 0, 3$.



Graph 1

Furthermore, unlike many other distributions, the Skew Normal has a closed-form expression for its moment generating function, $E(\exp(ay))$, a result which is needed in our analysis to obtain a closed-form solution for our model. We summarise our main result in the following proposition.

Proposition 1 *Under LinEx loss function and Skew Normal density, the distribution $n(y)$ of forecasts for which all forecasters have the same expected wages $\hat{w}$, is given by*

$$n\left(\hat{y}\right)=\frac{P \times 2\varphi\left(\hat{y}/\sigma\right)\Phi\left(\lambda\hat{y}/\sigma\right)}{\left(\hat{w}-v/b\right)+(s/ba^2)2\exp\left(\left(a^2\sigma^2/2\right)-a\hat{y}\right)\Phi\left(\lambda/\sqrt{1+\lambda^2}a\sigma_t\right)+(s/ab)\hat{y}-\left(s/a^2b\right)} \tag{2}$$

where a and λ are the loss and density asymmetry parameters respectively and φ (.) and Φ(.) are the standard normal density and cumulative density, respectively.

Proof See Appendix.

Corollary 1 *Proposition 1 contains the result of Laster et al. (1999) in equation (8) under normality, as a special case.*

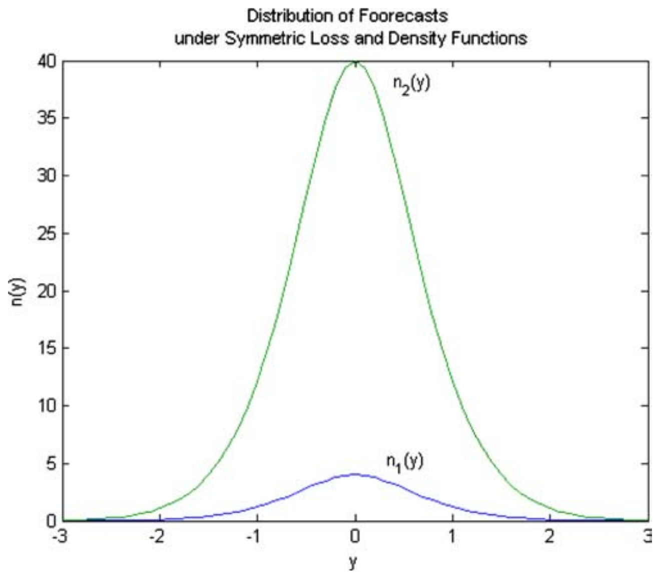
Proof As $a \to 0$ the LinEx loss reduces to the symmetric quadratic loss and as $\lambda \to 0$ the Skewed Normal reduces to the normal density function. Then equation (2) reduces to

$$n\left(\hat{y}\right) = \frac{P \times \varphi\left(\hat{y}/\sigma\right)}{\left(\hat{w} - v/b\right) + (s/b)\,\sigma^2 + (s/b)\,\hat{y}^2} \tag{3}$$

which is identical to equation (8) of Laster et al. under normality.

3 Trading-off preferences, densities and incentives

The implied distribution of forecasts in equation (2) now depends not only on the incentives parameters s and b, but also on loss and density function asymmetry parameters a and λ, respectively. Then, a trade-off between preferences, density and incentives parameters may be possible. A natural question that arises is the nature of the effect of a parameter change on the shape of the distribution function (2). Laster et al. have shown that under loss and density symmetry, the higher the ratio s/b the higher the concentration around the conditional mean. A natural way to study these properties would be to calculate the first four moments of $n\left(\hat{y}\right)$ and differentiate with respect to the parameters of interest, but one would obtain extremely complicated expressions. Alternatively, we adapt the numerical example of Laster et al. on page 304 to illustrate our arguments. Consider a standard normal density and parameter values $P = 5 \times 10^6$, $s_1/\mathrm{b} = 50 \times 10^3$ and $s_2/\mathrm{b} = 5 \times 10^3$, then from equation (3) we generate the following Graph 2. Clearly, irrespective of the value of s/b ratio, forecasts tend to cluster symmetrically around zero, which is the assumed mean of the density function and the optimal forecast under symmetric loss and density functions. Higher emphasis on the intensive forecast user loss, i.e. higher s/b ratio, results in smaller dispersion around that mean as Laster et al. indicate.



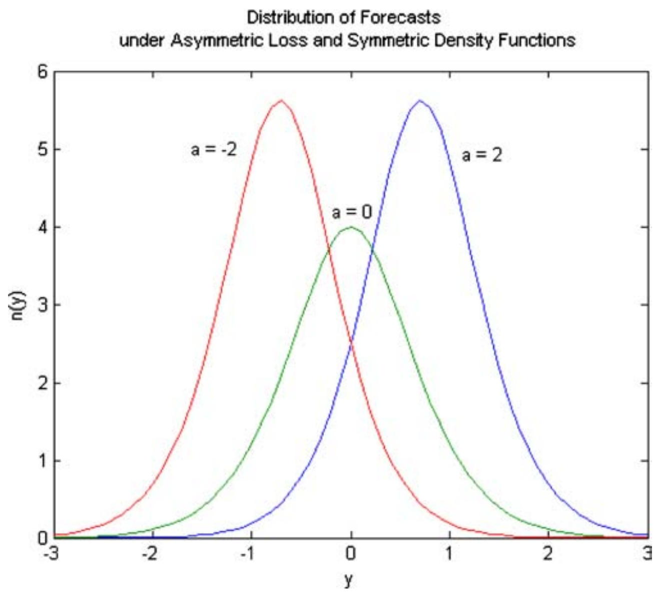
Graph 2

Assuming the existence of loss function asymmetry $a \neq 0$, we repeat our numerical illustrations assuming $a = 2$ and $a = -2$ but maintaining the assumption of density normality $\lambda = 0$ and $s_1 = 50 \times 10^3$. Clearly, for $a = 2$ the forecaster assigns higher penalty to under-prediction relative to over-prediction, thus resulting in a movement of the forecast distribution to the right of the density mean. In this case, the 'clustering' of forecasts that Laser et al. refer (1999, p. 304), moves from around the mean of the density function, towards around the optimal forecast under asymmetric loss. It is worth noting that the distribution of forecasts is no longer symmetric. Similarly, for $a = -2$ the forecaster assigns higher penalty to over-prediction relative to under-prediction, thus resulting in a movement of the forecast distribution to the left of the density mean. These results are depicted in Graph 3.

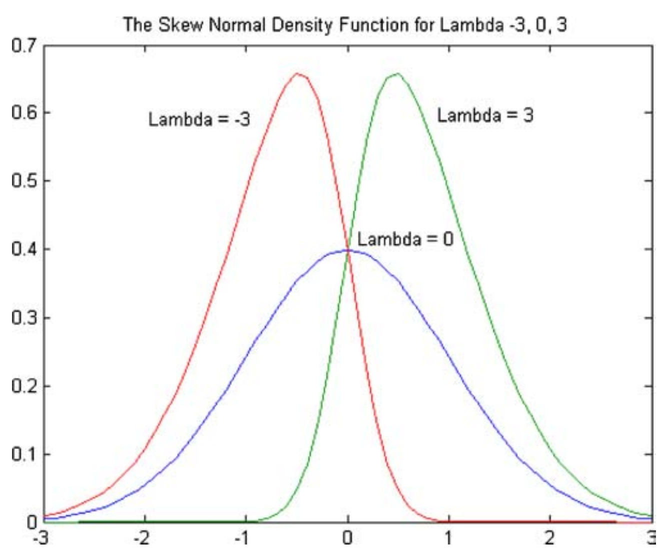
The effect of density function asymmetry on the distribution of forecasts can be seen by allowing for $\lambda \neq 0$, in particular $\lambda = 3$ and $\lambda = -3$, whilst keeping the loss function symmetric $\alpha = 0$ and $s_1 = 50 \times 10^3$. Our numerical results are depicted in Graph 4.

In Graph 4 we observe that the presence of density function asymmetries results in forecast distribution asymmetries in a similar fashion to loss function results in Graph 3. Positive density asymmetry, $\lambda > 0$, is consistent with $a > 0$, i.e. higher penalty to under-predictions relative to over-predictions. Likewise, negative density asymmetry, $\lambda < 0$, is consistent with $a < 0$, i.e. higher penalty to over-predictions relative to under-predictions. This result indicates that the joint presence of λ and α with opposite signs generate effects on the shape of forecast distribution that tend to offset each other.

We explore the empirical relevance of our results by considering empirical studies from the earnings forecasting literature. An established stylised fact is the well-known optimism bias in analysts' earnings forecasts. The literature has



Graph 3



Graph 4

recently addressed this issue with a sequence of empirical studies which intend to reveal the reasons behind this behaviour. Gu and Wu (2003) and Basu and Markov (2004) present evidence suggesting that the observed optimism bias is due to skewness in the forecast error distribution whilst the analysts' loss function may remain linear and symmetric. On the other side, Clatworthy et al. (2006) present evidence that such phenomena are attributed to loss function rather than density asymmetries. This is not entirely surprising given the reciprocity between preference and distribution asymmetries as shown in section 2.

4 Concluding remarks

Laster et al. (1999) built and economic model to show that the dispersion of the equilibrium distribution of forecasters depends on the relative importance given on the intensive forecast users' loss versus the publicity gain from occasional users. That is, the higher the weight on the forecast user loss, the higher the concentration of the forecasters around the mean of the distribution of the variable under prediction. These results depend heavily on the assumption of symmetric quadratic loss preferences for the forecast user and symmetric density for the variable under investigation. In this paper we examine the effects of generalising loss preferences and probability densities to allow for asymmetries through the LinEx loss and the Skewed Normal density, respectively. We derive the new equilibrium distribution of forecasts which contains the results of Laster et al. as a special case. The presence of asymmetric preferences is shown to cause a movement of the distribution away from the conditional mean, towards the optimal forecast under loss asymmetry. Furthermore, forecasts now tend to cluster around this quantity in an asymmetric way. These effects tend to be further strengthened or partially offset by the presence of skewness in the distribution of data, a result consistent with the conclusions of Christodoulakis (2005).

Appendix

Proof of Proposition 1 Let the data generating process be $y_t = \sigma_t v_t$, where $v_t \sim SN(\lambda)$ and λ is a real constant, with density function

$$\text{pdf}(v) = 2\varphi(v)\,\Phi(\lambda v)$$

where φ and Φ denote the standard normal density and cumulative density respectively. For a loss asymmetry parameter $a \in R\backslash\{0\}$, the LinEx loss function takes the form

$$L\left(y_{t+s} - \hat{y}_{t+s}\right) = \frac{1}{a^2}\left(\exp\left(a\left(y_{t+s} - \hat{y}_{t+s}\right)\right) - a\left(y_{t+s} - \hat{y}_{t+s}\right) - 1\right)$$

which under the Skew Normal density has expected value

$$E_t L\left(y_{t+s} - \hat{y}_{t+s}\right) = \frac{1}{a^2}\left(E_t \exp\left(a y_{t+s}\right)\exp\left(-a\hat{y}_{t+s}\right) + a\hat{y}_{t+s} - 1\right)$$

The expectation on the right side is the moment generating function of a Skew Normal variable, which from Proposition 1 of Arnold and Lin (2004) leads to

$$E_t L\left(y_{t+s} - \hat{y}_{t+s}\right) = \frac{2}{a^2}\exp\left(\frac{a^2\sigma_t^2}{2}\right)\Phi\left(\frac{\lambda}{\sqrt{1+\lambda^2}}a\sigma_t\right)\exp\left(-a\hat{y}_{t+s}\right) + \frac{1}{a}\hat{y}_{t+s} - \frac{1}{a^2}$$

Using the above expression and equation (4) of Laster et al. (1999), we obtain the distribution $n(y)$ of forecasts for which all forecasters have the same expected wage $\hat{w}$.

$$n\left(x\right)=\frac{P\times 2\varphi\left(y/\sigma\right)\Phi\left(\lambda y/\sigma\right)}{\left(\hat{w}-v/b\right)+\left(s/ba^{2}\right)2\exp\left(\left(a^{2}\sigma^{2}/2\right)-a\hat{y}\right)\Phi\left(\left(\lambda/\sqrt{1+\lambda^{2}}\right)a\sigma_{t}\right)+\left(s/ab\right)\hat{y}-\left(s/a^{2}b\right)}$$

□

References

Arnold, B.C., Lin, G.D.: Characterizations of the skew-normal and the generalized chi distributions. Sankhyã: Indian J Stat **66**(4), 593–606 (2004)

Basu, S., Markov, S.: Loss function assumptions in rational expectations tests on financial analysts' earnings forecasts. J Account Econ **38**, 171–203 (2004)

Christodoulakis, G.A.: Financial forecasts in the presence of asymmetric loss aversion, skewness and excess kurtosis. Finan Res Lett **2**, 227–233 (2005)

Christoffersen, P.F., Diebold, F.: Optimal prediction under asymmetric loss. Econ Theory **13**, 808–817 (1997)

Clatworthy, M., Peel, D., Pope, P.: Are analysts' loss functions asymmetric? Working Paper 2006/05, Lancaster University Management School, UK (2006)

Dolado, J.J., Maria-Dolores, R., Ruge-Murcia, F.J.: Nonlinear monetary policy rules: some evidence for the US. Working Paper, CEPR, London (2003)

Dolado, J.J., Maria-Dolores, R., Naveira, M.: Are monetary policy reaction functions asymmetric? The role of non-linearity in the Phillips curve. Eur Econ Rev **49(2)**, 485–503 (2005)

Granger, C.W.J.: Prediction with a generalized cost of error function. Operat Res Q **20**, 199–207 (1969)

Gu, Z., Wu, J.S.: Earnings skewness and analyst forecast bias. J account Econ **35**, 2–29 (2003)

Laster, D., Bennett, P., Geoum, I.S.: Rational bias in macroeconomic forecasts. Q J Econ **114**(1), 293–318 (1999)

Nobay, A.R., Peel, D.: Optimal discretionary monetary policy in a model of asymmetric central bank preferences. Econ J **113**, 657–665 (2003)

O'Hagan, A., Leonard, T.: Bayes estimation subject to uncertainty about parameter constraints. Biometrika **63**, 201–203 (1976)

Ruge-Murcia, F.: The inflation bias when the central banker targets the natural rate of unemployment. Eur Econ Rev **48**, 91–107 (2004)

Varian, H.: A Bayesian approach to real estate assessment. In: Feinberg, S.E., Zellner, A. (eds.) Studies in Bayesian economics in honour of L.J. Savage, pp. 195–208. Amsterdam: North-Holland (1974)